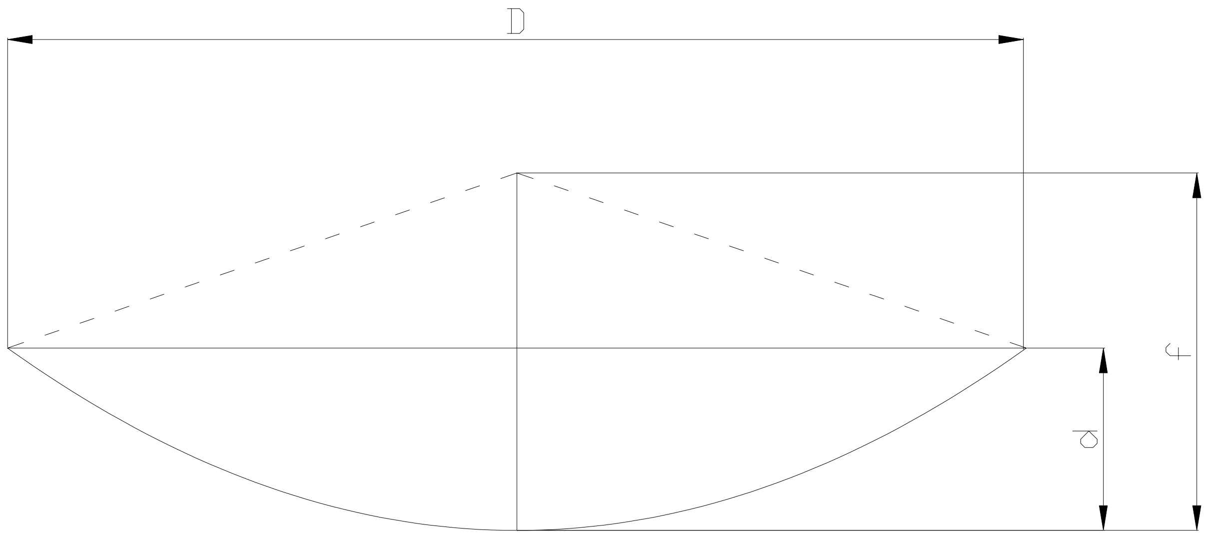
Calculation behind the design of Geodesic Parabolic Reflector Antenna (Rev.2)

Yoshiyuki Takeyasu / JA6XKQ

2024 December 29



D : Diameter

d : Depth of dish

f : Focal length

$$f = \frac{1}{4} \cdot \frac{\left(\frac{D}{2}\right)^2}{d}$$

$$d = \frac{1}{4} \cdot \frac{\left(\frac{D}{2}\right)^2}{f}$$

$$F = \frac{f}{D}$$

<u>Figure 1. Definition of dimension</u>

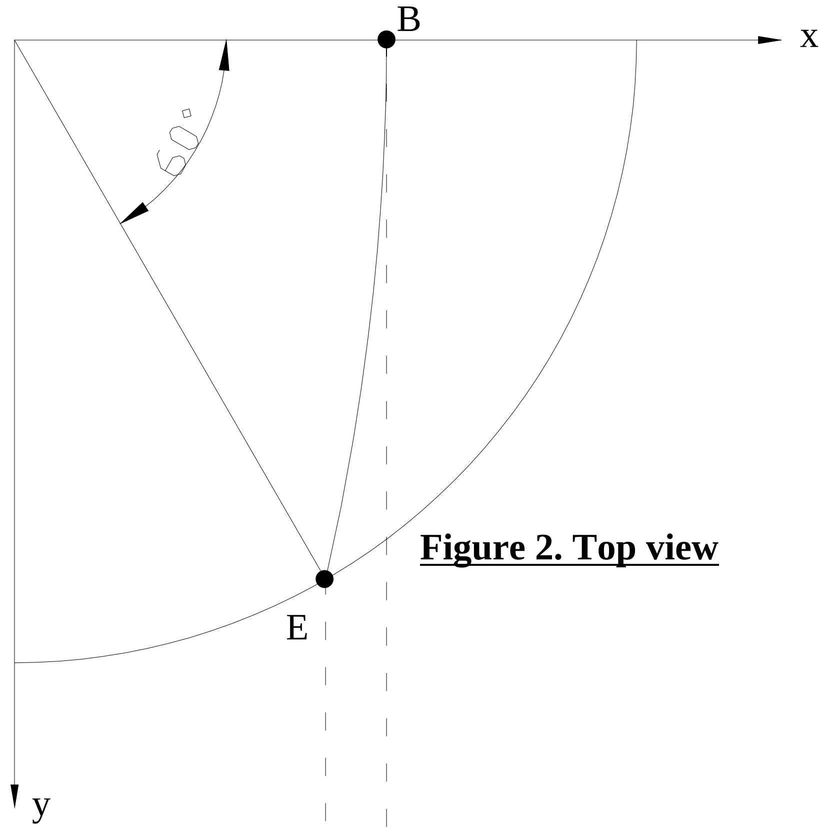


Figure 2. Top view

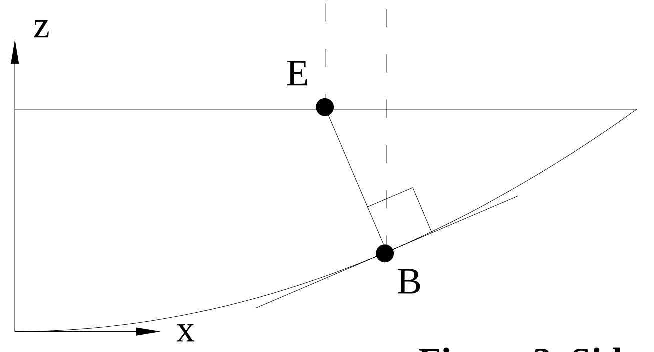


Figure 3. Side view

The Geodesic line **BE** in the top view is shown as a straight line **BE** in the side view. The first proposition is to determine the equation of the Geodesic line **BE** in the side view.

Since the Geodesic line **BE** in the side view intersects the tangent to the parabola perpendicularly at the point B and also passes through the point E, the equation of the straight line **BE** can be determined.

In other words, determining the equation is finding the point **B** that intersects the tangent perpendicularly.

Let the coordinates of point **B** and point **E** be (x_B, z_B) and (x_E, z_E), respectively.

The slope of the tangent to a parabola is expressed by the following equation, which is the first derivative of the parabola.

$$z = \frac{1}{4f} \cdot x^2$$

$$\frac{dz}{dx} = \frac{1}{4f} \cdot 2x = \frac{x}{2f}$$

Therefore, the slope at point **B** is:

$$\frac{x_B}{2f}$$

Since the Geodesic line **BE** is perpendicular to the tangent line, the slope of the Geodesic line **BE** is:

$$-\frac{2f}{x_B}$$

The slope of the Geodesic line **BE** is as above, and the line passes through the point **E**, so the equation is:

$$z - z_E = -\frac{2f}{x_B} \cdot (x - x_E)$$

To find the intersection **B**, solve the following simultaneous equations regarding the line **BE** and the parabola.

$$\begin{cases} z - z_E = -\dfrac{2f}{x_B} \cdot (x - x_E) \\ z = \dfrac{1}{4f} \cdot x^2 \\ x = x_B \\ z = z_B \end{cases}$$

Substituting z from the line equation into the parabola equation yields the following cubic equation for x_B.

$$\frac{1}{4f} \cdot {x_B}^2 - z_E = -\frac{2f}{x_B} \cdot (x_B - x_E)$$

$$\frac{1}{4f} \cdot {x_B}^3 - (z_E - 2f) \cdot x_B - 2f \cdot x_E = 0$$

$$where$$

$$x_E = \frac{D}{2} \cdot \cos\left(\frac{60}{180}\pi\right)$$

$$z_E = d$$

Find the root of this cubic equation for x_B using Excel’s Goal-seek tool.

Finally the equation of the Geodesic line **BE** in the side view is found.

$$z - z_E = -\frac{2f}{x_B} \cdot (x - x_E)$$

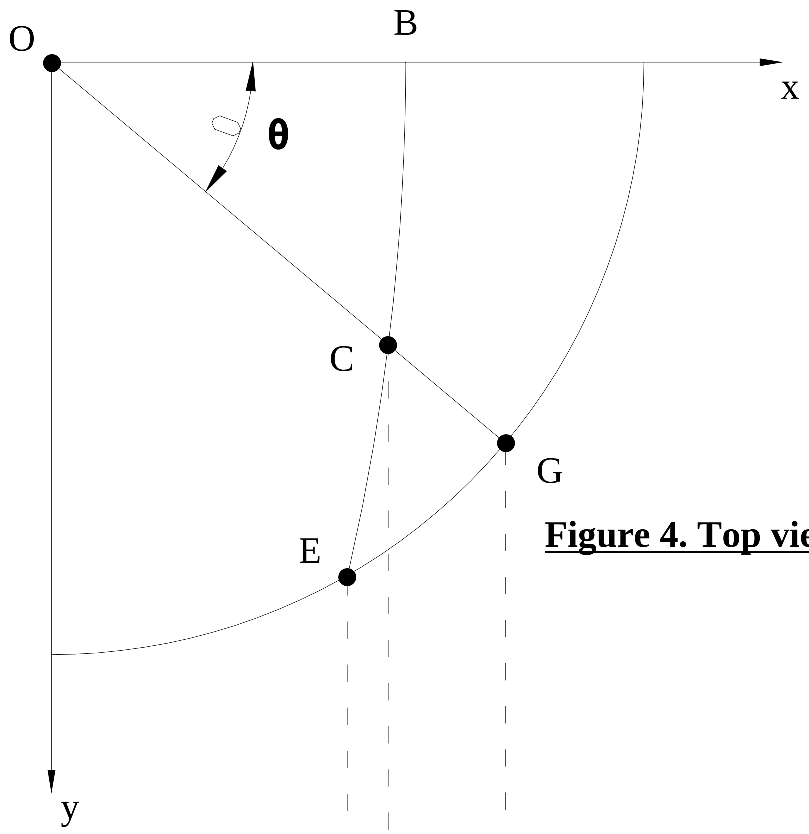


Figure 4. Top view

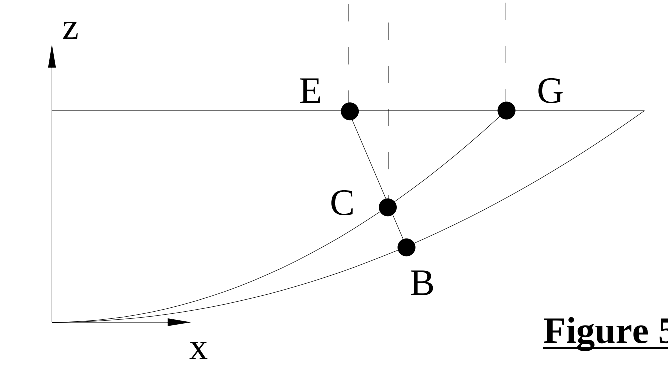


Figure 5. Side view

The next proposition is to find the coordinates of the Geodesic line **BE** in top view. To do this, subdivide the Geodesic line by assuming a radial rib **OG** rotated θ degrees from the x-axis in top view. The assumed radial rib **OG** intersects the Geodesic line at point **C**. The purpose is to find the coordinates of the point **C**.

The radial rib **OG** is a parabola in side view, and is expressed by the following equation.

$$z = \frac{1}{4f} \cdot \frac{1}{\left\{\cos\left(\frac{\theta}{180}\pi\right)\right\}^2} \cdot x^2$$

The coordinates of the intersection point **C** can be obtained by solving simultaneous equations of the already known equation on the Geodesic line **BE** and the above equation.

$$\begin{cases} z - z_E = -\dfrac{2f}{x_B} \cdot (x - x_E) \\ z = \dfrac{1}{4f} \cdot \dfrac{1}{\left\{\cos\left(\frac{\theta}{180}\pi\right)\right\}^2} \cdot x^2 \end{cases}$$

Substituting z from the line equation into the parabola equation yields the following quadratic equation for x.

$$ax^2 - kx + (kx_E - z_E) = 0$$

$$where$$

$$a = \frac{1}{4f} \cdot \frac{1}{\left\{\cos\left(\frac{\theta}{180}\pi\right)\right\}^2}$$

$$k = -\frac{2f}{x_B}$$

The roots of the quadratic equation are:

$$x = \frac{k \pm \sqrt{k^2 - 4a(kx_E - z_E)}}{2a}$$

One of these roots is the x-coordinate of point **C**, and by substituting the root into the above-mentioned simultaneous equations, the z-coordinate of point **C** can be found.

Since the x-coordinate of point C in top view is known in the above steps, the y-coordinate of point C is:

$$y_C = x_C \tan\left(\frac{\theta}{180}\pi\right)$$

Using the above procedure, the coordinates of point **C** in three-dimensional space were determined.

To obtain the complete Geodesic line **BE**, repeat the above procedure for variable **θ** from 0 to 60 degrees. In the original design, the variable **θ** was varied by 5 degrees. If the increments are made finer, the accuracy of the obtained Geodesic line **BE** will improve.

The length of the Geodesic line BE, divided and approximated by straight lines, is calculated as follows.

$$BE = \sum_{m=0}^{11} \sqrt{(x_{m+1} - x_m)^2 + (y_{m+1} - y_m)^2 + (z_{m+1} - z_m)^2}$$